CHAPTER 2

UNITS AND MEASUREMENTS

PHYSICAL QUANTITY: The quantities which can be measured directly or indirectly is called physical quantity**.**

Fundamental quantities : Quantities which cannot be expressed in terms of any other physical quantity

length, mass, time, temperature etc are fundamental quantities.

Derived quantities :Quantities that can be expressed in terms of fundamental quantities are called **derived quantities**. Area, volume, density etc. are examples for derived quantities.

Fundamental and derived units :
Unit of a physical quantity is defined as the **established standard** used for comparison of the given physical quantity.
The units in which the fundamental quantities are measured are called fundamental units and the units used to measure derived quantities are called derived units.

SYSTEM OF UNITS

S.I System.: In S.I system units there are 7 fundamental quantities and 2 supplementary physical quantity

Physical quantity	Unit	Symbol
Fundamental		
Length	metre	m
Mass	kilogram	kg
Time	second	s
Electric current	ampere	A
Temperature	kelvin	K
Luminous intensity	candela	cd
Amount of substance	mole	mol
Supplementary		
Plane angle	radian	rad
Solid angle	steradian	sr

The advantages of SI system

The SI standards do not vary with time as they are based on the properties of atoms.
SI system of units are coherent system of units, in which the units of derived quantities are obtained as multiples or submultiples of certain basic units.
It is a metric system.

It is rational system i.e., it gives one unit for one physical quantity, e.g., for energy of any type, i.e., mechanical or heat or electrical. There is only one unit, Joule (J) but in M.K.S. system unit for mechanical energy is Joule.

Units of large distances

(i) Light year
I light year = 9.4673×10^{15} m

(ii) Astronomical unit (AU)
1 AU = 1.496×10^{11} m

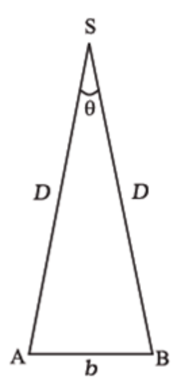
(iii) Parsec
1 parsec =3.08×10^{16} m

Measurement of large distances

Parallax is the shift in the position of an object when observed from two different positions.

To measure the distance of a far away planet

To measure the distance D of a far away planet S by the parallax method, we observe it from two different positions A and B. The $\angle ASB$ represented by symbol θ is called the parallax angle or parallactic angle.

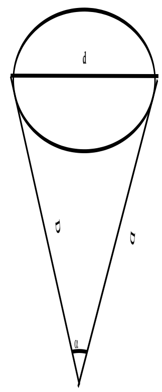


We approximately take AB as an arc of length b of a circle with centre at S and the distance D as the radius AS = BS so that AB = b = D θ where θ is in radians.

Measuring the size of a planet

If d is the diameter of the planet and α the angular size of the planet (the angle subtended by d at the earth), we have $\alpha = d/D$ The angle α can be measured from the same location on the earth.
Since D is known, the diameter d
of the planet can be determined

$\boxed{D = b/\theta}$



DIMENSIONS

Dimensions of a physical quantity are the powers to which the fundamental quantities must be raised.

Dimensions of fundamental physical quantities

Fundamental quantity	**Dimension**
Length	L
Mass	M
Time	T
Temperature	K
Electric current	A
Luminous intensity	cd
Amount of substance	mol

Dimensional formula- the expression which shows the relation of a physical quantity with the fundamental physical quantities is called dimensional formula.
Eg. The dimensional formula of volume is $[M^0 L^3 T^0]$

The dimensional equation of volume is V= $[M^0 L^3 T^0]$

Quantity	Formula	dimansion	Unit
Velocity	v= displacement/time	$[LT^{-1}]$	m/s
Acceleration	a= velocity /time	$[LT^{-2}]$	m/s^2
Momentum	P= mv	$[MLT^{-1}]$	kgm/s
Impulse	I = Ft	$[MLT^{-1}]$	Ns
Force	F = ma	$[ML^2T^{-2}]$	N
Pressure	P= F/A	$[M^1L^{-1} T^{-2}]$	Pa
Kinetic energy	K = ½ mv^2	$[ML^2 T^{-2}]$	J
Power	P= W/t	$[ML^2 T^{-3}]$	W
Density	D= m/V	$[ML^{-3}]$	kgm^{-3}
Moment of inertia	I = mr^2	$[ML^2]$	kgm^2

Principle of homogeneity of dimensions
An equation is dimensionally correct if the dimensions of the various terms on either side of the equation are the same. This is called the principle of homogeneity of dimensions.

Uses of dimensional analysis
(i) convert a physical quantity from one system of units to another.
(ii) check the dimensional correctness of a given equation.
(iii) establish a relationship between different physical quantities in an equation.

To check the dimensional correctness of a given equation using dimensional analysis .

Verify the dimensional correctness of a given equation : S= ut + (½)at^2.

By substituting dimension of the physical quantities in the above relation
S = [L]

[ut] = $[LT^{-1}][T] = [L]$

[1/2 at^2] = $[LT^{-2}][T^2] = [L]$

As in the above equation dimensions of each term on both sides are same, so this equation is dimensionally correct

Qn .check if the following equations are dimensionally correct

a. T= $2\pi\frac{\sqrt{L}}{g}$
b. E= ½ mv^2
c. λ = h/mv (h- Planck's constant, m - mass, v - velocity).

1. **Establish a relationship between different physical quantities in an equation.**

Qn. Find an expression for the time period T of a simple pendulum. The time period T

may depend upon (i) mass m of the bob (ii) length l of the pendulum and (iii) acceleration due to gravity g at the place where the pendulum is suspended.

Give : $T \propto m^x\ l^y\ g^z$

or $T = km^x\ l^y\ g^z$ ...(1)

where k is a dimensionless constant of proportionality. Rewriting equation (1) with dimensions,

$\left[T^1\right] = \left[M^x\right]\left[L^y\right]\left[LT^{-2}\right]^z$

Or, $\left[T^1\right] = \left[M^x\right]\left[L^{y+z}\right]\left[T^{-2z}\right]$

Comparing the powers of M, L and T on both sides, we get: x = 0, y + z = 0 and −2z = 1

Solving for x, y and z, x = 0, y = ½ and z = –½

From equation (1), $T = km^0\ l^{1/2}\ g^{-1/2}$

Or, $\boxed{T = k\sqrt{\frac{l}{g}}}$

Experimentally the value of k is determined to be 2π. $\therefore$ $\boxed{T = 2\pi\sqrt{\frac{l}{g}}}$

2. To convert a physical quantity from one system of units to another.

The measure of a physical quantity is nu = constant

If a physical quantity X has dimensional formula $[M^a L^b T^c]$ and if (derived) units of that physical quantity in two systems are $[M_1^a L_1^b T_1^c]$ and $[M_2^a L_2^b T_2^c]$ respectively and n1 and n2 be the numerical values in the two systems respectively, then

$$n_1[M_1^a L_1^b T_1^c] = n_2 [M_2^a L_2^b T_2^c]$$

$$\therefore n_2 = n_1\left[\frac{M_1}{M_2}\right]^a\left[\frac{L_1}{L_2}\right]^b\left[\frac{T_1}{T_2}\right]^c$$

Thus knowing the values of fundamental units in two systems and numerical value in one system, the numerical value in other system may be evaluated.

Qn. Given the value of G in cgs system is 6.67×10^{-8}dyne cm^2 g^{-2}. Calculate its value in SI units.

Given:

In cgs system	In SI system
$G_{cgs} = 6.67 \times 10^{-8}$	$G = ?$
$M_1 = 1g$	$M_2 = 1\ kg$
$L_1 = 1\ cm$	$L_2 = 1m$
$T_1 = 1s$	$T_2 = 1s$

The dimensional formula for gravitational constant is $[M^{-1}L^3T^{-2}]$.

In cgs system, dimensional formula for G is

$$[M_1^x L_1^y T_1^z]$$

In SI system, dimensional formula for G is

$$[M_2^x L_2^y T_2^z]$$

Here $x = -1$, $y = 3$, $z = -2$

$$G\left[M_2^x L_2^y T_2^z\right] = G_{cgs}\left[M_1^x L_1^y T_1^z\right]$$

$$G = G_{cgs}\left[\frac{M_1}{M_2}\right]^x \left[\frac{L_1}{L_2}\right]^y \left[\frac{T_1}{T_2}\right]^z$$

$$= 6.67 \times 10^{-8}\left[\frac{1\,g}{1\,kg}\right]^{-1} \left[\frac{1\,cm}{1\,m}\right]^3 \left[\frac{1\,s}{1\,s}\right]^{-2}$$

$$= 6.67 \times 10^{-8}\left[\frac{1\,g}{1000\,g}\right]^{-1} \left[\frac{1\,cm}{100\,cm}\right]^3 [1]^{-2}$$

$$= 6.67 \times 10^{-11}$$

In SI units,

$$G = 6.67 \times 10^{-11}\ \text{N m}^2\ \text{kg}^{-2}$$

Limitations of dimensional analysis

(i) The value of dimensionless constants cannot be determined by this method.
(ii) This method cannot be applied to equations involving exponential and trigonometric functions.
(iii) It cannot be applied to an equation involving more than three physical quantities.
(iv) It can check only whether a physical relation is dimensionally correct or not. It cannot tell whether the relation is absolutely correct or not.

Practice question :

1. The air bubble formed by explosion inside water perform oscillations with time period T which depends on pressure (*p*), density (*ρ*) and on energy due to explosion (*E*). Establish relation between T, *p, E* and *ρ.*
2. The volume of a liquid flowing out per second of a pipe of length l and radius r is written by a student as

   $$v = \frac{\pi}{8}\frac{Pr^4}{\eta l}$$

   where P is the pressure difference between the two ends of the pipe and η is coefficent of viscosity of the liquid having dimensional formula $ML^{-1}T^{-1}$. Check whether the equation is dimensionally correct.

Rules of Rounding Off

If the digit to be dropped is less than 5, then the preceding digit is left unchanged. e.g., 1.54 is rounded off to 1.5.

1. If the digit to be dropped is greater than 5, then the preceding digit is raised by one. e.g., 2.49 is rounded off to 2.5.
2. If the digit to be dropped is 5 followed by digit other than zero, then the preceding digit is raised by one. e.g., 3.55 is rounded off to 3.6.
3. If the digit to be dropped is 5 or 5 followed by zeros, then the preceding digit is raised by one, if it is odd and left unchanged if it is even. e.g., 3.750 is rounded off to 3.8 and 4.650 is rounded off to 4.6.

Significant figure

The number of meaningful digits in a number is called the number of significant figures.

Rules for finding significant figures

i) All the non−zero digits in a number are significant.
ii) All the zeroes between two non−zeroes digits are significant, irrespective of the decimal point.
iii) If the number is less than 1, the zeroes on the right of decimal point but to the left of the first non−zero digit are not significant. (In <u>0.0</u>2868 the underlined zeroes are not significant).
iv) The zeroes at the end without a decimal point are not significant. (In 23080 , the trailing zero is not significant).
v) The trailing zeroes in a number with a decimal point are significant. (The number 0.07100 has four significant digits).

Examples

i) 30700 has three significant figures.
ii) 132.73 has five significant figures.
iii) 0.00345 has three and
iv) 40.00 has four significant figures.

Significant Figures in Calculation

(1) The result of an addition or subtraction in the number having different precisions should be reported to the same number of decimal places as are present in the number having the least number of decimal places. The rule is illustrated by the following examples :

(i) 33.3 (has only one decimal place)

$$\begin{array}{r} 33.3 \\ 3.11 \\ +\ 0.313 \\ \hline 36.723 \end{array}$$

36.723 (answer should be reported to one decimal place)

Answer = 36.7

(ii) Add 17.35 kg, 25.8 kg and 9.423 kg.

Of the three measurements given, 25.8 kg is the least decimal point

$\therefore$ 17.35 + 25.8 + 9.423 = 52.573 kg

Correct to three significant figures, 52.573 kg is written as 52.6 kg

(2) The answer to a multiplication or division is rounded off to the same number of significant figures as is possessed by the least precise term used in the calculation. The rule is illustrated by the following example :

(i) Multiply 3.8 and 0.125 with due regard to significant figures.
$3.8 \times 0.125 = 0.475$
The least number of significant figure in the given quantities is 2.
Therefore the result should have only two significant figures. $\therefore$ $3.8 \times 0.125 = 0.475 = 0.48$

Questions for practice

State the number of significant figures in the following : (a) 0.007 m^2 (b) 2.64 10^{24} kg (c) 0.2370 g (d) 6.320 J (e) 6.032 N $m.^2$ (f) 0.0006032 m^2.

Find the number of significant figures in :a) 10001 b) 0.01500 c) 0.040

The length, breadth and thickness of a rectangular sheet of metal are 4.234 m, 1.005 m, and 2.01 cm respectively. Give the area and volume of the sheet to correct significant figures.

Errors in measurement

The uncertainty in the measurement of a physical quantity is called error. It is the difference between the true value and the measured value of the physical quantity.
Types of errors
Systematic Error:
These errors occur due to a certain pattern of the system. Sources of these errors are

(i) Instrumental error – this is due to the imperfection of design or calibration of instrument .
It can be reduced by using more accurate instruments or applying zero corrections if required .

(ii) Personal errors: Errors occurring due to human carelessness, lack of proper setting, taking down incorrect reading are called personal errors.

Note: Systematic errors can be minimized by improving experimental techniques ,selecting better instruments and removing personal bias as far as possible .
Systematic error can be reduced by taking a large number of observations and the arithmetic mean of these observations giving the best possible value of measured quantity .

Random Errors

Errors which occur at random with respect to sign and size are called **Random errors**.

These occur due to unpredictable fluctuations in experimental conditions like temperature, voltage supply, mechanical vibrations, unbiased personal errors etc.

Absolute error :
Absolute error in the measurement of a physical quantity is the magnitude of the difference between the true value and the measured value of the quantity.

Let a physical quantity be measured n times. Let the measured value be a1, a2, a3, ….. an. The arithmetic mean of these value is the true value .

$$a_{mean} = (a_1 + a_2 + a_3 + \ldots + a_n)/n$$

Mean absolute error : It is the arithmetic mean of the magnitudes of absolute errors in all the measurements of the quantity. It is represented by . a◎ Thus

$$\Delta a_{mean} = (|\Delta a_1| + |\Delta a_2| + |\Delta a_3| + \ldots + |\Delta a_n|) / n$$

What is relative error?

If the accuracy in measuring a quantity a is Δ a, then the relative error in x is given by

$$\Delta a_{mean} / a_{mean}$$

What is percentage error?

If the accuracy in measuring a quantity a is Δ a , then the percentage error in x is given by

$$(\Delta a_{mean} / a_{mean}) \times 100\%$$

Errors propagating during mathematical operations

i) addition/subtraction

When two quantities are added or subtracted, the absolute error in the final result is the sum of the **absolute errors** in the individual quantities.

i.e, $\boxed{\pm \Delta Z = \pm \Delta A \pm \Delta B}$

ii) multiplication/division

When two quantities are multiplied or divided, the **relative error** in the result is the sum of the relative errors in the multipliers

i.e. $\boxed{\frac{\Delta Z}{Z} = \left(\frac{\Delta A}{A}\right) + \left(\frac{\Delta B}{B}\right)}$

iii) Raising to powers

The **relative error** in a physical quantity raised to the power k is the k times the relative error in the individual quantity.

If $Z = \frac{A^p B^q}{C^r}$, then

$$\boxed{\frac{\Delta Z}{Z} = p\left(\frac{\Delta A}{A}\right) + q\left(\frac{\Delta B}{B}\right) + r\left(\frac{\Delta C}{C}\right)}$$

Qn. The refractive index of water is found to have values 1.29 1.33 1.34 1.35 1.32 1.36 1.30 and 1.33 calculate mean value absolute error relative error and percentage error

Mean value = Sum of all the observations/No. of observations.
∴ Mean value = (1.29 + 1.33 + 1.34 + 1.35 + 1.32 + 1.36 + 1.30 + 1.33)/8
= 10.62/8
= 1.3275
≈ 1.33

or absolute error in each term,

$x_1 = |1.33 - 1.29| = 0.04$

$x_2 = |1.33 - 1.33| = 0.00$

$x_3 = |1.33 - 1.34| = 0.01$

$x_4 = |1.33 - 1.35| = 0.02$

$x_5 = |1.33 - 1.32| = 0.01$

$x_6 = |1.33 - 1.36| = 0.03$

$x_7 = |1.33 - 1.30| = 0.03$

$x_8 = |1.33 - 1.33| = 0.00$

∴ Mean Absolute error = (0.04 + 0.00 + 0.01 + 0.02 + 0.01 + 0.03 + 0.03 + 0.00)/8

= 0.14/8

= 0.0175

≈ 0.018

∴ Relative Error = Mean absolute error/Mean Value

= 0.018/1.33

= 0.135

≈ 0.12

∴ % Error = 0.12 × 100

% Error = 12 %

Questions for practice :

The resistance R = V/I where (100 ± 5)V and I = (10 ± 0.2)A. Find the percentage error in R.

Find the relative error **Z, if** $Z = \frac{A^4 B^{1/3}}{CD^{3/2}}$.

The period of oscillation of a simple pendulum is $= 2\pi\sqrt{\frac{L}{g}}$. Measured value of L is 20.0 cm known to 1 mm accuracy and time for 100 oscillations of the pendulum is found to be 90 s using a wrist watch of 1 s resolution. What is the accuracy in the determination of g ?